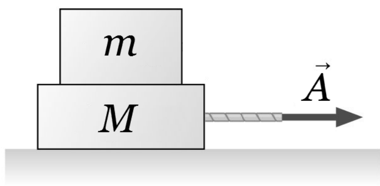
g Magnitude of Free Fall Acceleration

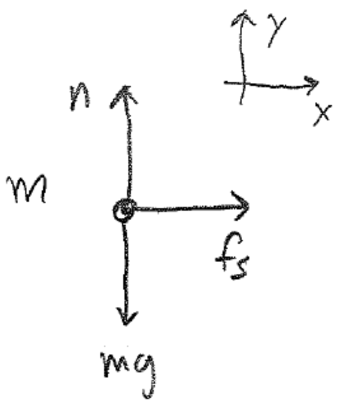
Unless otherwise directed, drag should be neglected.

Any integrals in free-response problems must be evaluated. Questions about magnitudes will state so explicitly.

I. (16 points) A block of mass M is at rest on a frictionless horizontal surface. A block of mass m is at rest on top of it. The coefficient of static friction between the blocks is μ_s, and the coefficient of kinetic friction is μ_k. What maximum horizontal applied force magnitude A can be applied to the block of mass M, if the block of mass m is not to slide? Express your answer in terms of other parameters defined in the problem, and physical or mathematical constants. *(On Earth.)*



Use Newton's Second Law. First, analyze the top block, of mass m. Sketch a Free Body Diagram. The block has a normal force magnitude n upward, a gravitational force magnitude mg downward, and a static frictional force magnitude f_s to the right. Choose a coordinate system. It is easier if you choose one for which the known acceleration is along one axis. Write out Newton's Second Law for each dimension. I'll show signs explicitly, so symbols represent magnitudes.



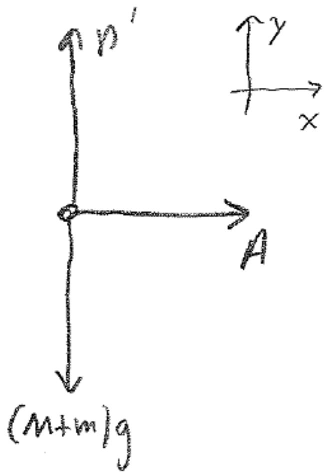
$$\sum F_y = n - mg = ma_y = 0 \qquad \Rightarrow \qquad n = mg$$

$$\sum F_x = f_s = ma_x$$

Since we're looking for the *maximum* force magnitude A, the top block will have its maximum acceleration, so the static friction force will have its maximum value. Therefore

$$f_s = \mu_s n = \mu_s mg \qquad \text{so} \qquad \mu_s mg = ma_x \qquad \Rightarrow \qquad a_x = \mu_s g$$

If the top block doesn't slide, both blocks have the same acceleration, and can be analyzed as a single object. Again, sketch a Free Body Diagram. The combined object has a normal force magnitude n' upward, a gravitational force magnitude $(M+m)\,g$ downward, and an applied force magnitude A to the right. Again, choose a coordinate system, making things easier by letting one axis point in the direction of the known acceleration.

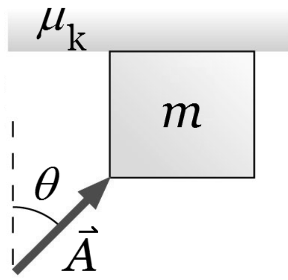


$$\sum F_x = A = (M+m)\,a_x$$

As the acceleration of the top block must be the same as that of the combined object, substitute the expression found for a_x.

$$A = (M+m)\,\mu_s g$$

II. (16 points) An applied force of magnitude A, directed at an angle θ from the vertical, acts on a block of mass m sliding to the right along a horizontal ceiling, as shown. If the coefficient of kinetic friction between the block and the ceiling is μ_k, what is the magnitude of the block's acceleration? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. *(On Earth.)*



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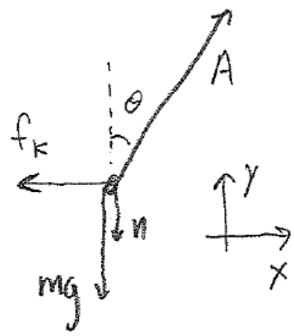
Use Newton's Second Law. Sketch a Free Body Diagram. The block has a normal force magnitude n downward, a gravitational force magnitude mg downward, a kinetic frictional force magnitude f_k to the left, and the applied force magnitude A. Choose a coordinate system. It is easier if you choose one for which the known acceleration is along one axis. Write out Newton's Second Law for each dimension. I'll show signs explicitly, so symbols represent magnitudes.

$$\sum F_y = A_y - n - mg = ma_y = 0 \quad \Rightarrow \quad A\cos\theta - n - mg = 0 \quad \Rightarrow \quad n = A\cos\theta - mg$$

$$\sum F_x = A_x - f_k = ma_x \quad \Rightarrow \quad A\sin\theta - \mu_k n = ma_x \quad \Rightarrow \quad A\sin\theta - \mu_k\left(A\cos\theta - mg\right) = ma_x$$

Solve for a_x.

$$a_x = \frac{A\sin\theta - \mu_k\left(A\cos\theta - mg\right)}{m} = \frac{A}{m}\left(\sin\theta - \mu_k\cos\theta\right) + \mu_k g$$



1. (6 points) If it can be determined in the problem above, what is the direction of the block's acceleration?

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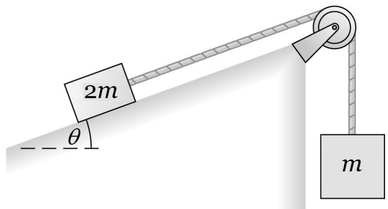
Although the block is sliding to the right, without actual numeric values, it cannot be determined whether its speed is increasing or decreasing.

The direction of the acceleration cannot be determined from the information provided.

III. (16 points) The block of mass $2m$ is at rest on a frictionless plane that makes an angle θ with the horizontal. It is attached to an ideal rope that travels parallel to the plane, then over an ideal pulley, to a hanging block of mass m. What is the angle θ? Express your answer in terms of other parameters defined in the problem, and physical or mathematical constants. *(On Earth.)*



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Use Newton's Second Law. First, analyze the block of mass m. Sketch a Free Body Diagram. The block has a tension force magnitude T upward, and a gravitational force magnitude mg downward. Choose a coordinate system. Since the acceleration is zero, I'll choose one so no forces need be resolved into components. Write out Newton's Second Law. I'll show signs explicitly, so symbols represent magnitudes.

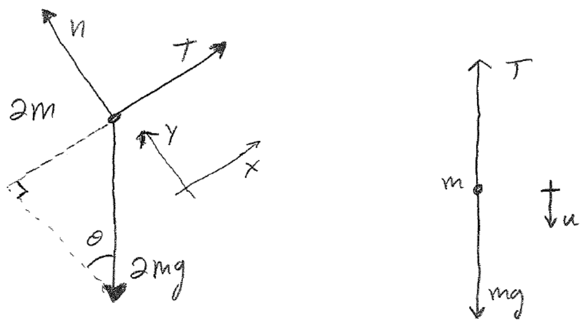
$$\sum F_u = mg - T = ma_u = 0 \qquad \Rightarrow \qquad T = mg$$

Next, analyze the block of mass $2m$. Again, sketch a Free Body Diagram. This block has a normal force magnitude n up perpendicular to the plane, a gravitational force magnitude $2mg$ downward, and a tension force magnitude T up parallel to the plane. Choose a coordinate system. Since the acceleration is zero, I'll choose one to minimize the number of forces that must be resolved into components. Again, write out Newton's Second Law.

$$\sum F_x = T - 2mg_z = 2ma_x = 0 \qquad \Rightarrow \qquad T = 2mg\sin\theta \qquad \Rightarrow \qquad \theta = \sin^{-1}\left(\frac{T}{2mg}\right)$$

Substitute the expression found for T.

$$\theta = \sin^{-1}\left(\frac{mg}{2mg}\right) = \sin^{-1}\left(\frac{1}{2}\right)$$



2. (6 points) In the problem above, the tension in the rope is T. The block of mass $2m$ is now given a nudge so it begins to slide up the plane. Describe the tension T' in the rope while the block slides (but after the "nudge"). *(On Earth.)*

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Once the "nudge" is over and the blocks are moving, all the same forces are acting as before the nudge. The velocity will still be constant (although no longer the constant zero), so the acceleration is still zero (i.e., the blocks are still in equilibriums, although no longer in static equilibrium). The force analysis is exactly the same, so

$$T' = T$$